

# Damping Rates and Mean Free Paths of Soft Fermion Collective Excitations in a Hot Fermion-Gauge-Scalar Theory\*

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We study the transport coefficients, damping rates and mean free paths of soft fermion collective excitations in a hot fermion-gauge-scalar plasma with the goal of understanding the main physical mechanisms that determine transport of chirality in scenarios of non-local electroweak baryogenesis. The focus is on identifying the *different* transport coefficients for the different branches of *soft* collective excitations of the fermion spectrum. These branches correspond to collective excitations with opposite ratios of chirality to helicity and different dispersion relations. By combining results from the hard thermal loop (HTL) resummation program with a novel mechanism of fermion damping through heavy scalar decay, we obtain a robust description of the different damping rates and mean free paths for the soft collective excitations to leading order in HTL and lowest order in the Yukawa coupling. The space-time evolution of wave packets of collective excitations unambiguously reveals the respective mean free paths. We find that whereas both the gauge and scalar contribution to the damping rates are *different* for the different branches, the *difference* of mean free paths for both branches is mainly determined by the decay of the heavy scalar into a hard fermion and a soft collective excitation. We argue that these mechanisms are robust and are therefore relevant for non-local scenarios of baryogenesis either in the Standard Model or extensions thereof.

## I. INTRODUCTION

One of the fundamental problems confronting particle astrophysics is that of baryogenesis, the origin of the abundance of matter over antimatter in the observed Universe. The Standard Model (SM) [1] and extensions thereof offer the tantalizing possibility of providing an explanation for baryogenesis with physics at the electroweak scale (for a description of mechanisms of baryogenesis see Ref. [2]). Although the Standard Model satisfies the three criteria for baryogenesis: (i) violation of baryon number, (ii) violation of  $C$  and  $CP$ , and (iii) departure from thermal equilibrium [3–5], a wealth of theoretical evidence combined with experimental bounds on the  $CP$  violating phases in the Kobayashi-Maskawa (KM) matrix, and the current bounds on the Higgs mass,  $m_H \geq 88$  GeV, from LEP2 [6] and the most recent lattice simulations [7] seem to lead to the conclusion that the minimal Standard Model [8] may not be able to explain the observed baryon asymmetry but perhaps extensions could naturally lead to such an explanation (for recent reviews see Refs. [9,10]).

An important non-equilibrium ingredient for a successful scenario of electroweak baryogenesis is a strongly first order phase transition that proceeds via the nucleation of the true (broken symmetry) vacuum in the background of the false (unbroken symmetry) vacuum plus non-trivial topological field configurations, the sphalerons, that are responsible for the baryon violating processes [9].

Amongst the several proposals for explaining the baryon asymmetry, we focus on transport aspects related to non-local or charge transfer baryogenesis [4]. This scenario assumes a strongly first order phase transition in the electroweak theory (or extensions thereof) [9,10]. As the nucleated bubbles of the broken phase in the background of the unbroken (false vacuum) phase expand, a flux of fermions from the unbroken phase scatter off the bubble walls,  $CP$  violating interactions near the wall are converted to an asymmetry in the baryon number via sphaleron processes in the unbroken phase [9,8].

An important ingredient in this scenario is the transport properties of chiral fermions in the regime of small spatial momentum [4,8] but in a *plasma* at temperature  $T \approx 100$  GeV. In particular there are two different regimes for

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non-local baryogenesis depending on whether the mean free path of the quarks is smaller or larger than the width of the bubble walls [11].

The hot plasma modifies dramatically the properties of fermions [13–15], in particular when  $gT \gg M$  with  $g$  the gauge coupling constant and  $M$  the zero temperature fermion mass, the fermion spectrum is characterized by two branches of *collective excitations*, one branch corresponds to the in-medium renormalized fermion, with a positive ratio of chirality to helicity, and the other branch is a novel collective excitation in the medium, the plasmino, with a negative ratio of chirality to helicity [13,14]. The two branches have a gap  $M_{eff} \approx gT$  corresponding to an effective fermion mass that does not break chiral symmetry [13–15].

The fermion and plasmino branches describe excitations with different chirality properties and therefore these are bound to play an important role in charge transfer (non-local) mechanisms of baryogenesis [9,8]. For soft momenta  $k \leq gT$  these branches are very different from each other, while for large momenta  $k \geq gT$  they approach the usual fermion dispersion relation, and the amplitude of plasmino excitations (the wave function renormalization) vanishes exponentially fast [13–18]. Whereas dispersion relations of soft collective excitations in the plasma within the context of baryogenesis had been studied in Ref. [8] and extended to describe neutralinos in the medium in supersymmetric extensions in Ref. [19], the damping or thermalization rate has only been studied at zero spatial momentum, i.e., the collective excitations are at rest with respect to the plasma [20] or for hard spatial momentum [21]  $k \geq T$ .

Early attempts to estimate the diffusion coefficient made use of Boltzmann equations and a series of approximations including a simplification of the gluon propagator [11,12]. Some of those results were reproduced in [21] for quarks of *hard* momentum,  $k \geq T$ .

The two limits that were previously studied, zero spatial momentum and very large spatial momentum ( $k \geq T$ ), *do not* reveal important features of the collective excitations which are only present at non-zero but soft momenta  $0 < k \leq gT$  where the dispersion relations are different [13–15]. At zero spatial momentum, both branches of the fermionic dispersion relation, i.e., fermions and plasminos coincide and therefore the damping rates for both collective excitations must be the same. In the opposite limit  $k \geq T$ , the fermion branch approaches the dispersion relation of an ordinary vacuum fermion losing all features of the collective behavior, and the plasmino branch decouples from the spectrum because its wave function renormalization vanishes exponentially.

In this article we present a detailed study of the relaxation rates (damping or thermalization rates) and mean free paths for *soft* collective excitations with non-vanishing momenta  $k \leq gT$  in a theory that includes both gauge and Yukawa interactions. To our knowledge such study had not been pursued before within the context of gauge and scalar (Yukawa) interactions. Furthermore, since the different collective excitations have different group velocities, it is important to understand in detail their mean free paths. This is very relevant for non-local baryogenesis [4,8], since the mean free path determines the attenuation length for the transport of chirality by the different collective excitations. As emphasized above, the different branches correspond to collective excitations with *opposite* ratios of chirality to helicity, and different dispersion relations.

Our *main observations* in this article are that the transport properties of these two different collective excitations for soft momenta will in general be *different*. Since these excitations carry chirality, different damping or relaxation rates will result in *different mean free paths for collective excitations carrying opposite chirality*. As has been argued repeatedly in the literature [4,8,9] the transport of chirality plays a fundamental role in non-local baryogenesis, thus a consistent microscopic study of baryogenesis in these scenarios must necessarily address the transport of chirality by the soft collective excitations and is the motivation for our study.

An important aspect of soft excitations in a plasma is that their consistent description requires a non-perturbative resummation scheme, the hard thermal loop (HTL) program [22]. For soft excitations with  $k \leq gT$  and vacuum masses  $\ll gT$  (with  $g$  the generic gauge coupling constant) the leading corrections from the thermal self-energies in the high temperature limit are of  $\mathcal{O}(gT)$ , hence of the same order as the tree level terms in the propagators. The leading HTL correction is determined by the hard momentum region in the self-energy loop [14,16,22]. However, the damping rate of soft collective excitations require that the fermion and gauge bosons propagators and vertices in the self-energy be HTL resummed [23–26]. The HTL resummation rendered infrared finite the damping rate of collective excitations at rest in the plasma [23], however the damping rate of *moving* collective excitations is infrared divergent arising from the exchange of transverse gauge bosons [24]. Whereas in QCD (and in the SU(2) sector of the electroweak theory) the putative non-perturbative magnetic mass provides an infrared cutoff [24,25], in QED (and certainly the U(1) sector of the electroweak theory) such a possibility is unavailable and the infrared divergence remains [24,25]. The fermion propagator in QED has been studied within the eikonal (Bloch-Nordsieck) approximation in *real time* [26]. This analysis revealed an anomalous relaxation that explained the nature of the infrared divergences in the abelian theory and determined that the relaxation time scale of fermionic collective excitations  $\propto [g^2 T \ln 1/g]^{-1}$  [26]. This anomalous relaxation was recently reproduced via different methods in scalar QED [27] that has the same leading HTL structure as QCD and QED for the fermion and gauge boson propagators [28]. Although there is no magnetic screening mass in the abelian theory, the relaxation scales obtained by Refs. [26,27] are *similar* to those obtained in QCD [24,25].

More recently we have obtained the damping rate for fermions in a fermion-scalar plasma and pointed out that the decay of a heavy scalar (Higgs) into fermion pair results in a quasiparticle width for the fermion, i.e., the scalar decay leads to a fermion damping rate [29]. In this article we combine the results obtained in QCD and QED for the damping rate of the collective excitations with the contribution from the scalars to provide a detailed understanding of the relaxation scales and mean free paths for the soft chirality carrying collective excitations in the plasma.

**The goals:** In this article we study in detail the damping rates and mean free paths of soft collective excitations in a plasma of gauge, fermion and scalar particles as befits the scenarios for electroweak baryogenesis. The focus is to provide a quantitative study of the *different* relaxation scales for the soft collective excitations with opposite ratios of chirality to helicity.

However, rather than studying these aspects in the full Standard Electroweak Model or any particular extension of it, we focus on the understanding of the robust and generic features of the damping rates and mean free paths of soft collective excitations in an abelian plasma of fermions, gauge bosons and scalars with simple Yukawa couplings. The main reasons why the results of this simplification *are relevant* to the physics of the electroweak theory, are that:

(i) The structure of hard thermal loops is similar in the abelian and non-abelian theories [13–16,18,23,24] with the differences only in the gauge group factors and strength of couplings. This is manifestly explicit in similar damping rates obtained for QCD and QED [24,26]. A very complete study of the dispersion relations for collective excitations in the electroweak plasma has been provided in Ref. [8]. The results of that work clearly show that the HTL contributions to the self-energies are universal in the sense that the strong, weak and Higgs sectors yield contributions that are similar in form and only differ in the appropriate gauge group structure factors.

(ii) The simple physics that leads to the damping of fermionic excitations via the decay of a heavy scalar [29] is fairly robust and transcends a particular model, it is mainly described by the kinematics of the decay of a heavy scalar into fermion pair in the medium.

To model the set of parameters of the electroweak theory we assume a scalar of mass  $m \approx 100$  GeV according with the current LEP2 bounds on the Higgs mass [6], and a first order phase transition temperature  $T \approx 100$  GeV which results in  $m \approx T$ . This will become important in the analysis of the scalar contribution to the fermion self-energy. Furthermore we will focus on the lightest quarks and leptons for which  $gT \gg M$  with  $M$  being the fermion vacuum mass. Only for the lightest fermions we expect soft collective excitations in the medium, since for fermions with  $M \gg gT$  the in-medium corrections to the self-energy are perturbatively small for weak couplings. For these lightest generations, the Yukawa couplings  $y \ll g$  and the scalar contributions to the fermionic self-energies are perturbative. However, as we will see in detail below, the scalar contribution to the damping rates and mean free paths is important.

**Main Results:** The main results can be summarized with the statement that for soft collective excitations that carry opposite ratios of chirality to helicity, i.e., fermions and plasminos, the damping rates  $\Gamma_{\pm}(k)$  and mean free paths  $\lambda_{\pm}(k)$  are *different*. The difference in these relaxation scales is maximal in the range of momenta  $0 < k \leq gT$ . The difference in the mean free paths of the soft collective modes is dominated by the scalar contribution that we explicitly compute in Eq. (3.36). The gauge boson contribution to the damping rate of moving collective excitations arises mainly from the absorption and emission of soft transverse gauge bosons for which a hard thermal loop resummation of the propagators and vertices is required. On the other hand the scalar contribution to the damping rates arises from the *decay* of the scalar into a hard single particle excitation and a soft collective excitation. Both contributions are different for the different branches, in the gauge case the difference is determined by the different group velocities.

We also provide a wave packet analysis to extract the mean free paths of the moving collective excitations. We find that the *difference* of mean free paths for the collective excitations with different ratios of chirality to helicity is mainly determined by the *scalar* contribution to the damping rates. The mean free paths and their difference are explicitly given by Eqs. (4.9-4.10).

The article is organized as follows. In Sec. II we introduce the renormalized, real-time Dirac equation for fermions in the medium, the renormalization aspects are important for consistency because we need to isolate the wave function renormalization factors for the collective excitations. The main reason for studying the effective in-medium Dirac equation in *real time* is that this formulation allows us to extract unambiguously the mean free paths or attenuation lengths by focusing on the time evolution of wave packets of collective excitations. This treatment bypasses the assumptions of transport by diffusion. Furthermore, in this manner we can make direct contact with the results of Refs. [26,27,29] and adapt them to the situation under study. In this section we obtain the different contributions to the self-energy and renormalize the ultraviolet divergences. Sec. III is devoted to a detailed study of the damping rates and we establish some of our main results. In Sec. IV we study the time evolution of wave packets of collective excitations and extract the mean free paths or attenuation lengths. We then show that the main contribution to the *difference* between the mean free paths for the two soft collective excitations arises from the scalar contribution to the damping rate. Sec. V summarizes our results and describes their implications for the standard model (or its extensions) and points out the possible modifications to our results in extensions of the Standard Model that include light scalars.

## II. EFFECTIVE DIRAC EQUATION IN THE MEDIUM

An unambiguous identification of the damping rates requires an analysis of the retarded propagators in the complex frequency plane. The lifetimes (damping rates) are identified as complex poles in the unphysical Riemann sheet.

Rather than computing the self-energies in imaginary time and perform the analytic continuations to obtain the retarded self-energies in the complex frequency plane, we obtain the effective Dirac equation directly in *real time*. This alternative approach has two main advantages: (i) it allows us to make contact with the real-time results for the relaxation of the single particle Green's function presented in Refs. [26,27], (ii) by following the space-time evolution of an initially prepared wave packet of collective excitations we can extract directly the mean free paths.

As mentioned in the introduction we consider an abelian theory with one fermion species and one real scalar coupled to the fermion via a Yukawa interaction. We will work in the Coulomb gauge in which  $\nabla \cdot \vec{A} = 0$  because the HTL self-energies have a more clear interpretation in this gauge.

The self-energies in leading order in HTL have been proven to be the same in Coulomb gauge and in covariant and axial gauges [18,23,24]. Ref. [18] provides a discussion of the gauge parameter independence of the HTL contribution to the self-energies.

The Lagrangian density is given by

$$\begin{aligned} \mathcal{L} = & \bar{\Psi} \left( i \not{\partial} - g_0 \gamma^0 A^0 + g_0 \vec{\gamma} \cdot \vec{A}_T - y_0 \phi - M_0 \right) \Psi + \bar{\Psi} \eta + \bar{\eta} \Psi \\ & + \frac{1}{2} \left[ \left( \partial^\mu \vec{A}_T \right)^2 + \left( \nabla A^0 \right)^2 + \left( \partial^\mu \phi \right)^2 - m_0^2 \phi^2 \right] + \mathcal{L}_I[\phi] , \end{aligned} \quad (2.1)$$

where the Grassmann valued source terms were introduced to obtain the effective Dirac equation in the medium by analyzing the linear response to these sources [29]. We have not included the possible gauge couplings of the scalar since these will not contribute to leading order in the hard thermal loops and lowest order in the Yukawa couplings.

The self-interaction of the scalar field accounted for by the term  $\mathcal{L}_I[\phi]$  need not be specified to lowest order.

We now write the bare fields  $\Psi$ ,  $\phi$ ,  $\vec{A}_T$ ,  $A^0$  and  $\eta$  in terms of the renormalized quantities (hereafter referred to with a subscript  $r$ ) by introducing the renormalization constants:

$$\begin{aligned} \Psi &= Z_\psi^{1/2} \Psi_r , \quad \phi = Z_\phi^{1/2} \phi_r , \quad \vec{A}_T = Z_A^{1/2} \vec{A}_{rT} , \quad A^0 = Z_A^{1/2} A_r^0 , \quad \eta = Z_\psi^{-1/2} \eta_r , \\ y &= y_0 Z_y^{-1} Z_\phi^{1/2} Z_\psi , \quad g = g_0 Z_g^{-1} Z_A^{1/2} Z_\psi , \\ m_0^2 &= [\delta_m(T) + m^2(T)] Z_\phi^{-1} , \quad M_0 = (\delta_M + M) Z_\psi^{-1} . \end{aligned} \quad (2.2)$$

With the above definitions,  $\mathcal{L}$  can be expressed as:

$$\begin{aligned} \mathcal{L} = & \bar{\Psi}_r \left( i \not{\partial} - g \gamma^0 A_r^0 + g \vec{\gamma} \cdot \vec{A}_{rT} - y \phi_r - M \right) \Psi_r + \frac{1}{2} \left[ \left( \partial^\mu \vec{A}_{rT} \right)^2 + \left( \nabla A_r^0 \right)^2 + \left( \partial^\mu \phi_r \right)^2 - m^2(T) \phi_r^2 \right] \\ & + \mathcal{L}_{rI}[\phi_r] + \bar{\eta}_r \Psi_r + \bar{\Psi}_r \eta_r + \mathcal{L}_{\text{c.t.}} , \end{aligned} \quad (2.3)$$

where  $m(T)$ ,  $M$ ,  $g$  and  $y$  are the renormalized parameters, and the counter-term Lagrangian  $\mathcal{L}_{\text{c.t.}}$  is given by

$$\begin{aligned} \mathcal{L}_{\text{c.t.}} = & \bar{\Psi}_r \left( i \delta_\psi \not{\partial} - g \delta_g \gamma^0 A_r^0 + g \delta_g \vec{\gamma} \cdot \vec{A}_{rT} - y \delta_y \phi_r - \delta_M \right) \Psi_r \\ & + \frac{1}{2} \left[ \delta_A \left( \partial^\mu \vec{A}_{rT} \right)^2 + \delta_A \left( \nabla A_r^0 \right)^2 + \delta_\phi \left( \partial^\mu \phi_r \right)^2 - \delta_m(T) \phi_r^2 \right] + \delta \mathcal{L}_{rI}[\phi_r] . \end{aligned} \quad (2.4)$$

The terms with the coefficients

$$\begin{aligned} \delta_\psi &= Z_\psi - 1 , \quad \delta_\phi = Z_\phi - 1 , \quad \delta_A = Z_A - 1 , \\ \delta_M &= M_0 Z_\psi - M , \quad \delta_m(T) = m_0^2 Z_\phi - m^2(T) , \\ \delta_y &= Z_y - 1 , \quad \delta_g = Z_g - 1 , \end{aligned} \quad (2.5)$$

and  $\delta \mathcal{L}_{rI}[\phi_r]$  are the counterterms to be determined consistently in the perturbative expansion by choosing a renormalization prescription. The main reason for going through the renormalization is to extract the wave functions corresponding to the collective excitations since these will be important for the calculation of the damping rates. By choosing the wave function renormalization  $Z_\psi$  to be defined on-shell at zero temperature, we isolate the finite temperature from the zero temperature renormalizations unambiguously and recognize the wave function renormalization for the collective excitations directly.

We have included the finite temperature corrections to the scalar mass in the renormalized Lagrangian and explicitly introduced a finite temperature counterterm. The scalar mass receives finite temperature corrections also from hard thermal loops, proportional to  $T^2$ . As it will become clear below we are interested in the kinematic decay of the Higgs into the fermionic collective modes which will contribute to their mean free paths. By including the finite temperature corrected Higgs mass in the Lagrangian allows to treat the kinematic decay in terms of the in-medium corrected mass directly. The counterterm  $\delta_m(T)$  will be chosen consistently to cancel the finite temperature corrections to the mass. There is a small caveat, however, we envisage our analysis to be valid both in the (homogeneous) unbroken and the broken symmetry phases in the case of a strongly first order phase transition. The mass term in the Lagrangian should then be defined as the position of the pole of the finite temperature propagator for the Higgs. We will implicitly use this definition of the Higgs mass in what follows. Furthermore in our analysis we will *assume* that in either case, unbroken or broken symmetry phases the effective Higgs mass is large enough that its kinematic decay into the soft fermionic collective excitations is allowed, in particular we *assume* that  $m(T) \gg gT$ . In the broken symmetry phase and for a strong first order phase transition  $m^2(T) \approx m^2(0)$  with  $m(0) \approx T \approx 100$  GeV this assumption seems reasonable, in the unbroken phase whether this condition is satisfied depends on the details and strength of the first order phase transition.

If the effective mass in either phase does not allow the kinematical decay of the in-medium Higgs into soft fermionic collective modes then there is no Higgs contribution to their mean free path.

Estimates based on the finite temperature effective potential would seem to suggest that the Higgs mass may not be much heavier than  $gT_c$  [30,31], but conclusions based on effective potentials in gauge theories are fraught with ambiguities such as the gauge dependence of the effective potential which translates into gauge dependence of off-shell quantities, and the fact that the effective potential becomes complex [32–34] near its minimum. A Higgs mass defined as the second derivative of the effective potential at its minimum is not an on-shell quantity [34] (since the effective potential is defined at zero four momentum), only the position of the pole of the Higgs propagator is an on-shell and therefore gauge invariant quantity. Given the uncertainties on the scalar sector of the Standard Model, only a sound *experimental* bound can validate or rule out the assumption used in this work on the value of the Higgs mass.

We must also point out that our analysis is valid in the unbroken or the broken symmetry phases (under the proviso of the kinematic constraint for the decay of the Higgs) but certainly *not* near the wall of a nucleating bubble. The nature of the soft fermionic collective excitations in the presence of a wall is not well understood, since the hard thermal loop resummation that leads to the collective excitations is only valid in an homogeneous equilibrium background. Thus before we can extrapolate our analysis to the case of collective fermionic excitations near a bubble wall a consistent study of the effects of the wall on these collective modes must be pursued. As mentioned above our goal is to provide a quantitative analysis of the mean free paths of the soft fermionic collective modes at least in the homogeneous phases, with particular attention to the *difference* of mean free paths of the different collective modes.

The dynamics of expectation values and correlation functions of the quantum field is obtained by implementing the Schwinger-Keldysh closed-time-path formulation of non-equilibrium quantum field theory [35–37]. The main ingredient in this formulation is the *real time* evolution of an initially prepared density matrix and its path integral representation. It requires a path integral defined along a closed time path contour. This formulation has been described elsewhere within many different contexts and we refer the reader to the literature for details [35–37].

The effective Dirac equation is the equation of motion for the expectation value of the fermion field  $\psi(\vec{x}, t) \equiv \langle \Psi_r(\vec{x}, t) \rangle_\eta$  in the presence of the sources  $\bar{\eta}, \eta$ . Since the details and methods to obtain the effective Dirac equation in the medium have been described at length elsewhere [29], we refer the reader to Refs. [29,36,37] and simply quote the final result:

$$\left[ \left( i\gamma_0 \frac{\partial}{\partial t} - \vec{\gamma} \cdot \vec{k} - M \right) + \delta_\psi \left( i\gamma_0 \frac{\partial}{\partial t} - \vec{\gamma} \cdot \vec{k} \right) - \delta_M \right] \psi_{\vec{k}}(t) + \int_{-\infty}^t dt' \Sigma_{\vec{k}}(t-t') \psi_{\vec{k}}(t') = -\eta_{r\vec{k}}(t) , \quad (2.6)$$

where

$$\psi_{\vec{k}}(t) \equiv \int d^3x e^{-i\vec{k} \cdot \vec{x}} \psi(\vec{x}, t) .$$

and  $\Sigma_{\vec{k}}(t-t')$  is the total self-energy, which to lowest order is the sum of the transverse, longitudinal and Yukawa contributions respectively, i.e.,  $\Sigma_{\vec{k}}(t-t') = \Sigma_{\vec{k}}^T(t-t') + \Sigma_{\vec{k}}^L(t-t') + \Sigma_{\vec{k}}^Y(t-t')$ . The scalar contribution can be found in [29], and for the gauge contribution to the fermion self-energy the main ingredients are the real-time propagators

$$\begin{aligned} \langle A_{rTi}^{(a)}(\vec{x}, t) A_{rTj}^{(b)}(\vec{x}', t') \rangle &= -i \int \frac{d^3k}{(2\pi)^3} \mathcal{G}_{ij}^{ab}(\vec{k}; t, t') e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} , \\ \mathcal{G}_{ij}^{++}(\vec{k}; t, t') &= \mathcal{P}_{ij}(\vec{k}) [\mathcal{G}_{\vec{k}}^>(t, t') \Theta(t-t') + \mathcal{G}_{\vec{k}}^<(t, t') \Theta(t'-t)] , \end{aligned}$$

$$\begin{aligned}
\mathcal{G}_{ij}^{--}(\vec{k}; t, t') &= \mathcal{P}_{ij}(\vec{k}) [\mathcal{G}_k^>(t, t')\Theta(t' - t) + \mathcal{G}_k^<(t, t')\Theta(t - t')] , \\
\mathcal{G}_{ij}^{+-}(\vec{k}; t, t') &= \mathcal{P}_{ij}(\vec{k}) \mathcal{G}_k^<(t, t') , \\
\mathcal{G}_{ij}^{-+}(\vec{k}; t, t') &= \mathcal{P}_{ij}(\vec{k}) \mathcal{G}_k^>(t, t') , \\
\mathcal{G}_k^>(t, t') &= \frac{i}{2k} \left[ (1 + \tilde{n}_k) e^{-ik(t-t')} + \tilde{n}_k e^{ik(t-t')} \right] , \\
\mathcal{G}_k^<(t, t') &= \frac{i}{2k} \left[ \tilde{n}_k e^{-ik(t-t')} + (1 + \tilde{n}_k) e^{ik(t-t')} \right] , \\
\tilde{n}_k &= \frac{1}{e^{\beta k} - 1} , \quad \beta = \frac{1}{T} , \quad \mathcal{P}_{ij}(\vec{k}) = \delta_{ij} - \frac{k_i k_j}{k^2} .
\end{aligned} \tag{2.7}$$

The retarded time evolution must be understood as an initial value problem [29,36,37], which is set up consistently by taking the source to be switched on adiabatically from  $t = -\infty$  and switched off at  $t = 0$  thus providing the following initial conditions to the mean field

$$\psi_{\vec{k}}(t = 0) = \psi_{\vec{k}}(0) , \quad \dot{\psi}_{\vec{k}}(t < 0) = 0 . \tag{2.8}$$

It is convenient to separate the memory integral in (2.6) from  $t' = -\infty$  to  $t' = 0$  and  $t' = 0$  to  $t' = t$ , and for this purpose we introduce the kernel  $\sigma_{\vec{k}}(t - t')$  defined by the relation

$$\frac{d}{dt'} \sigma_{\vec{k}}(t - t') = \Sigma_{\vec{k}}(t - t') . \tag{2.9}$$

Since  $\eta_{r\vec{k}}(t > 0) = 0$ , we integrate by parts the memory kernel and finally obtain the equation of motion for  $t > 0$  in the following form

$$\left[ \left( i\gamma_0 \frac{\partial}{\partial t} - \vec{\gamma} \cdot \vec{k} - M \right) + \delta_\psi \left( i\gamma_0 \frac{\partial}{\partial t} - \vec{\gamma} \cdot \vec{k} \right) + \sigma_{\vec{k}}(0) - \delta_M \right] \psi_{\vec{k}}(t) - \int_0^t dt' \sigma_{\vec{k}}(t - t') \dot{\psi}_{\vec{k}}(t') = 0 . \tag{2.10}$$

The equation of motion (2.10) can now be solved by Laplace transform as befits an initial value problem. The Laplace transformed equation of motion is given by

$$\left[ i\gamma_0 s - \vec{\gamma} \cdot \vec{k} - M + \delta_\psi \left( i\gamma_0 s - \vec{\gamma} \cdot \vec{k} \right) - \delta_M + \sigma_{\vec{k}}(0) - s\tilde{\sigma}_{\vec{k}}(s) \right] \tilde{\psi}_{\vec{k}}(s) = [i\gamma_0 + i\delta_\psi \gamma_0 - \tilde{\sigma}_{\vec{k}}(s)] \psi_{\vec{k}}(0) , \tag{2.11}$$

where  $\tilde{\psi}_{\vec{k}}(s)$  and  $\tilde{\sigma}_{\vec{k}}(s)$  are the Laplace transforms of  $\psi_{\vec{k}}(t)$  and  $\sigma_{\vec{k}}(t)$  respectively:

$$\tilde{\psi}_{\vec{k}}(s) \equiv \int_0^\infty dt e^{-st} \psi_{\vec{k}}(t) , \quad \tilde{\sigma}_{\vec{k}}(s) \equiv \int_0^\infty dt e^{-st} \sigma_{\vec{k}}(t) .$$

Renormalization proceeds by requiring that the counterterms cancel the divergent parts of the self-energy.

Specifically, the renormalized self-energy contributions are given by

$$\begin{aligned}
\tilde{\Sigma}_{r\vec{k}}(s) &= \sigma_{r\vec{k}}(0) - s \tilde{\sigma}_{r\vec{k}}(s) , \\
\sigma_{r\vec{k}}(0) &= \sigma_{\vec{k}}(0) - \delta M - \delta_\psi \vec{\gamma} \cdot \vec{k} , \quad \tilde{\sigma}_{r\vec{k}}(s) = \tilde{\sigma}_{\vec{k}}(s) - i\gamma_0 \delta_\psi .
\end{aligned} \tag{2.12}$$

The counterterms will be displayed explicitly below. We thus obtain the fully renormalized effective Dirac equation in the medium, which is given by

$$\left[ i\gamma_0 s - \vec{\gamma} \cdot \vec{k} - M + \tilde{\Sigma}_{r\vec{k}}(s) \right] \tilde{\psi}_{\vec{k}}(s) = [i\gamma_0 - \tilde{\sigma}_{r\vec{k}}(s)] \psi_{\vec{k}}(0) , \tag{2.13}$$

The solution to (2.13) is therefore given by

$$\tilde{\psi}_{\vec{k}}(s) = \frac{1}{s} \left[ 1 + S(s, \vec{k}) \left( \vec{\gamma} \cdot \vec{k} + M - \tilde{\Sigma}_{r\vec{k}}(0) \right) \right] \psi_{\vec{k}}(0) , \tag{2.14}$$

where we have introduced the full renormalized fermion propagator in terms of the Laplace variable  $s$

$$S(s, \vec{k}) = \left[ i\gamma_0 s - \vec{\gamma} \cdot \vec{k} - M + \tilde{\Sigma}_{r\vec{k}}(s) \right]^{-1} . \tag{2.15}$$

The retarded propagator is obtained via the analytic continuation  $s = -i\omega + 0^+$ . Stable excitations correspond to isolated poles of  $S(s = -i\omega + 0^+, k)$  in the physical Riemann sheet in the complex  $\omega$  plane, whereas resonances correspond to complex poles in the second (or higher) Riemann sheet. These resonances are quasiparticles with lifetimes determined by the imaginary part of the complex pole and the spectral density for the fermion propagator features a Breit-Wigner form with a narrow peak in the weak coupling limit.

## A. The self-energy

Our main goal is to provide a description of damping and transport of soft fermionic collective excitations with typical momenta  $k \leq gT$ . In this region of soft momenta, the hard thermal loop contribution from gauge bosons must be treated non-perturbatively since it is of order  $gT$ . In principle if there are HTL contribution from the scalars, it should also be treated non-perturbatively. However we focus our study on the light quarks and leptons for which the typical Yukawa couplings  $y \ll g$ , the HTL contribution from the gauge bosons gives the only non-perturbative contribution (at least to lowest order).

Using the non-equilibrium Green's functions for free fields given in Ref. [29] and Eqs. (2.7), we find to one loop order the following contributions to the fermion self-energy:

$$\Sigma_k^y(t-t') = i\gamma_0 \Sigma_k^{y(0)}(t-t') + \vec{\gamma} \cdot \vec{k} \Sigma_k^{y(1)}(t-t') + \Sigma_k^{y(2)}(t-t') , \quad (2.16)$$

$$\Sigma_k^T(t-t') = i\gamma_0 \Sigma_k^{T(0)}(t-t') + \vec{\gamma} \cdot \vec{k} \Sigma_k^{T(1)}(t-t') + \Sigma_k^{T(2)}(t-t') , \quad (2.17)$$

$$\Sigma_k^L(t-t') = g^2 \int \frac{d^3q}{(2\pi)^3} \frac{1-2\bar{n}_q}{2\bar{\omega}_q(\vec{k}+\vec{q})^2} (\vec{\gamma} \cdot \vec{q} - M) \delta(t-t') , \quad (2.18)$$

where  $\Sigma_k^{y(i)}(t-t')$  to one loop order  $\mathcal{O}(y^2)$  are given in Ref. [29]. Although the transverse self-energy is available in the literature in the *imaginary time* formulation of finite temperature field theory, their form in *real time* is not readily available. It is given by

$$\begin{aligned} \Sigma_k^{T(0)}(t-t') &= g^2 \int \frac{d^3q}{(2\pi)^3 \bar{\omega}_{k+q}} \\ &\times \left[ \cos[(\bar{\omega}_{k+q} + \bar{\omega}_q)(t-t')](1 + \tilde{n}_{k+q} - \bar{n}_q) + \cos[(\bar{\omega}_{k+q} - \bar{\omega}_q)(t-t')](\tilde{n}_{k+q} + \bar{n}_q) \right] , \end{aligned} \quad (2.19)$$

$$\begin{aligned} \Sigma_k^{T(1)}(t-t') &= g^2 \int \frac{d^3q}{(2\pi)^3 \bar{\omega}_q \bar{\omega}_{k+q}} \left[ \frac{\vec{k} \cdot (\vec{k} + \vec{q}) \vec{q} \cdot (\vec{k} + \vec{q})}{k^2 (\vec{k} + \vec{q})^2} \right] \\ &\times \left[ \sin[(\bar{\omega}_{k+q} + \bar{\omega}_q)(t-t')](1 + \tilde{n}_{k+q} - \bar{n}_q) - \sin[(\bar{\omega}_{k+q} - \bar{\omega}_q)(t-t')](\tilde{n}_{k+q} + \bar{n}_q) \right] , \end{aligned} \quad (2.20)$$

$$\begin{aligned} \Sigma_k^{T(2)}(t-t') &= -g^2 M \int \frac{d^3q}{(2\pi)^3 \bar{\omega}_q \bar{\omega}_{k+q}} \\ &\times \left[ \sin[(\bar{\omega}_{k+q} + \bar{\omega}_q)(t-t')](1 + \tilde{n}_{k+q} - \bar{n}_q) - \sin[(\bar{\omega}_{k+q} - \bar{\omega}_q)(t-t')](\tilde{n}_{k+q} + \bar{n}_q) \right] , \end{aligned} \quad (2.21)$$

with  $\bar{\omega}_q = \sqrt{M^2 + \vec{q}^2}$ ,  $\bar{\omega}_{k+q} = \sqrt{(\vec{k} + \vec{q})^2}$ ,  $\bar{n}_q = (e^{\beta \bar{\omega}_q} + 1)^{-1}$  and  $\tilde{n}_{k+q} = (e^{\beta \bar{\omega}_{k+q}} - 1)^{-1}$  being the respective energies and distribution functions for the fermion and the gauge boson in the loop.

These contributions to the self-energy are solely in terms of the bare propagators for the internal lines in the loop. For the gauge contribution to the self-energy this is only valid for loop momenta  $k \gg gT$ , this is the region of the loop integral that gives rise to the HTL non-perturbative part. The region of soft loop momenta  $k \leq gT$  requires the HTL dressing of the internal propagators and vertices [23,24,26] and will be discussed in detail later.

For the scalar contribution in principle one should consider the different regions of the loop momentum, however, since the scalar is taken to represent a Higgs of mass  $\approx 90 - 100 \text{ GeV} \approx T$  the scalar line does not require HTL resummation. However as argued above, the scalar self-energy is perturbatively small as compared to the HTL part of the gauge boson self-energy. Its real part will yield a perturbatively small correction to the dispersion relation of the collective excitations and will be neglected, however the *imaginary* part will lead to a contribution to the damping rate, which is important since in lowest order HTL the collective excitations are stable. We anticipate now and it will be argued more forcefully below, that for a Higgs mass of  $\mathcal{O}(T)$  the energy conservation constraint for the imaginary part requires that for soft external momentum of the fermion, the *internal* fermion line must carry a momentum at least of  $\mathcal{O}(T)$  and therefore in order to obtain the scalar contribution to the damping rate the internal fermion line *does not* need HTL resummation. This is a very important point to which we will come back below and it results in that the scalar contribution does not suffer from the infrared sensitivity associated with the gauge boson contribution to the self-energy.

Before we discuss each contribution to the self-energy in detail, we proceed to fix the counterterms to render the effective Dirac equation finite to one loop order.

The ultraviolet-divergent parts of  $\sigma_{\vec{k}}(0)$  and  $\tilde{\sigma}_{\vec{k}}(s)$  entering in the Dirac equation (2.11) are contained in the zero temperature contributions. We use dimensional regularization in three spatial dimensions, introduce a renormalization scale  $\mathcal{K}$  and find

$$\begin{aligned}\sigma_{\vec{k}}(0) &= - \left[ \left( \frac{y^2}{16\pi^2} + \frac{g^2}{8\pi^2} \right) \vec{\gamma} \cdot \vec{k} - \left( \frac{y^2}{8\pi^2} - \frac{g^2}{2\pi^2} \right) M \right] \frac{\mathcal{K}^{-\epsilon}}{\epsilon} + \text{finite} , \\ \tilde{\sigma}_{\vec{k}}(s) &= -i\gamma_0 \left( \frac{y^2}{16\pi^2} + \frac{g^2}{8\pi^2} \right) \frac{\mathcal{K}^{-\epsilon}}{\epsilon} + \text{finite} ,\end{aligned}\tag{2.22}$$

where the finite parts contain the finite temperature contributions plus zero temperature finite terms. The counterterms can then be chosen in minimal subtraction to be given by

$$\delta_\psi = - \left( \frac{y^2}{16\pi^2} + \frac{g^2}{8\pi^2} \right) \frac{\mathcal{K}^{-\epsilon}}{\epsilon} , \quad \delta_M = M \left( \frac{y^2}{8\pi^2} - \frac{g^2}{2\pi^2} \right) \frac{\mathcal{K}^{-\epsilon}}{\epsilon} .\tag{2.23}$$

A convenient choice corresponds to on-shell renormalization at zero temperature, since in this case the residue at the poles of the propagators yield directly the wave function renormalizations of the collective excitations.

### III. THE IN-MEDIUM FERMION PROPAGATOR

After renormalization the zero temperature part of the self-energy is finite and perturbatively small, therefore we can neglect it and focus on the finite temperature part.

As mentioned in the introduction, our goal is to describe the transport of chirality by the soft collective excitations. Since the typical size of a non-perturbative gauge field configuration is  $\mathcal{O}(1/g^2T)$  [9], the contributions to baryon violating processes from quarks and leptons with soft momenta  $\leq gT$  is important [8]. For these soft fermionic excitations, the HTL contribution to the self-energy given by the hard momentum region of the loop integral (i.e.,  $k \geq T$ ) is  $\mathcal{O}(gT)$  and must be treated non-perturbatively.

Furthermore in the region near the electroweak phase transition with  $T \approx 100$  GeV and  $g \approx 0.3 - 0.6$  one finds that  $gT \gg M$  and the vacuum mass term can be neglected for the lightest leptons and quarks.

Therefore: (i) we neglect the zero temperature contribution, (ii) neglect the (renormalized) fermion mass, (iii) separate the HTL non-perturbative contribution from the hard gauge boson exchange from the perturbative contribution of soft gauge boson exchange and scalar exchange. Thus the gauge contribution to the fermion self-energy is separated into the hard thermal loop part  $\delta\tilde{\Sigma}_{r\vec{k}}(s)$  which is given by the hard loop momentum region  $k \approx T$ , and a correction  $^*\tilde{\Sigma}_{r\vec{k}}(s)$  [24] which arises from the soft region of the loop integration  $k \leq gT$ . In this region the internal fermions and gauge bosons as well as the vertices must be HTL resummed and a detailed analysis finds that this contribution is of order  $g^2T$  [24] (see below). Whereas the HTL contribution only gives an imaginary part below the light cone (Landau damping), the  $g^2T$  correction gives a contribution to the damping rate [24] both at rest and for a moving fermion. Since the Yukawa coupling for the lightest quarks and leptons is  $y \approx 10^{-4} \ll g$  and the ratio of the scales that enter in the scalar loop is  $m/T \approx 1$ , the scalar contribution will be treated perturbatively, along with that of  $^*\tilde{\Sigma}_{r\vec{k}}(s)$ .

Thus to this order, the renormalized full inverse fermion propagator is

$$S^{-1}(s, \vec{k}) = i\gamma_0 s - \vec{\gamma} \cdot \vec{k} + \delta\tilde{\Sigma}_{r\vec{k}}(s) + ^*\tilde{\Sigma}_{r\vec{k}}(s) + \tilde{\Sigma}_{r\vec{k}}^y(s) .\tag{3.1}$$

It is convenient to write the full fermion propagator in terms of a non-perturbative and a perturbative part

$$\begin{aligned}S^{-1}(s, \vec{k}) &= S_{NP}^{-1}(s, \vec{k}) + S_P^{-1}(s, \vec{k}) , \\ S_{NP}^{-1}(s, \vec{k}) &= i\gamma_0 s - \vec{\gamma} \cdot \vec{k} + \delta\tilde{\Sigma}_{r\vec{k}}(s) , \\ S_P^{-1}(s, \vec{k}) &= ^*\tilde{\Sigma}_{r\vec{k}}(s) + \tilde{\Sigma}_{r\vec{k}}^y(s) .\end{aligned}\tag{3.2}$$

The non-perturbative term will determine the position of the poles, i.e., the dispersion relation of the quasiparticles. In the HTL approximation it is known that the poles correspond to stable collective excitations, because the imaginary part of the self-energy is non-vanishing only below the light cone (Landau damping). Therefore the damping rate for on-shell excitations vanishes to leading order in HTL. The real part of the perturbative contribution will provide a small shift to the dispersion relation, whereas the imaginary part will provide the damping rate of the fermionic collective excitations on-shell. Because the correction to the dispersion relations from the perturbative part is small, we will neglect it but instead focus on the imaginary part since it will determine the damping rate which vanishes to leading order in the HTL.

Since the fermion self-energy is a sum of the contribution of the gauge bosons and that of the scalars, we study each contribution separately.



### A. Non-perturbative contribution to the fermion self-energy: hard thermal loops and collective excitations

We now focus on  $S_{NP}^{-1}(s, \vec{k})$ , which is given by the bare propagator plus the HTL contribution from the gauge boson exchange. The HTL approximation is obtained by considering the region  $q \approx T$  in the loop integrals, i.e., the loop momentum is hard and the external momentum is soft  $k \leq gT$ .

The HTL contribution to the fermion self-energy is obtained directly from the Laplace transform of Eqs. (2.19-2.21) in the HTL limit. In terms of the Laplace variable  $s$ , it is found to be given by

$$\delta\tilde{\Sigma}_{r\vec{k}}(s) = -\frac{g^2 T^2}{16k} \left\{ \gamma_0 \ln \left( \frac{is+k}{is-k} \right) + \vec{\gamma} \cdot \hat{k} \left[ 2 - \frac{is}{k} \ln \left( \frac{is+k}{is-k} \right) \right] \right\} . \quad (3.3)$$

The analytic continuation of  $\delta\tilde{\Sigma}_{r\vec{k}}(s)$  in the complex  $s$ -plane reads

$$\delta\tilde{\Sigma}_{r\vec{k}}(s = -i\omega \pm 0^+) \equiv \delta\Sigma_{\vec{k}}(\omega) = \delta\Sigma_{R\vec{k}}(\omega) \pm i\delta\Sigma_{I\vec{k}}(\omega) , \quad (3.4)$$

where

$$\delta\Sigma_{R\vec{k}}(\omega) = -\frac{g^2 T^2}{16k} \left\{ \gamma_0 \ln \left| \frac{\omega+k}{\omega-k} \right| + \vec{\gamma} \cdot \hat{k} \left[ 2 - \frac{\omega}{k} \ln \left| \frac{\omega+k}{\omega-k} \right| \right] \right\} , \quad (3.5)$$

$$\delta\Sigma_{I\vec{k}}(\omega) = \frac{\pi g^2 T^2}{16k} \left[ \gamma_0 - \vec{\gamma} \cdot \hat{k} \frac{\omega}{k} \right] \Theta(k^2 - \omega^2) , \quad (3.6)$$

and  $\hat{k} = \vec{k}/k$ .

To leading order in  $g$  (HTL), the full fermion propagator is given by  $*S^{-1}(\omega, \vec{k}) = \omega\gamma_0 - \vec{\gamma} \cdot \vec{k} + \delta\Sigma_{\vec{k}}(\omega)$  which can be written in the form [14,18]

$$*S(\omega, \vec{k}) = \frac{1}{2} \left[ * \Delta_+(\omega, \vec{k})(\gamma_0 - \vec{\gamma} \cdot \hat{k}) + * \Delta_-(\omega, \vec{k})(\gamma_0 + \vec{\gamma} \cdot \hat{k}) \right] , \quad (3.7)$$

where

$$* \Delta_{\pm}^{-1}(\omega, \vec{k}) = \omega \mp k - \frac{M_{eff}^2}{k} \left[ \left( 1 \mp \frac{\omega}{k} \right) Q_0 \left( \frac{\omega}{k} \right) \pm 1 \right] , \quad (3.8)$$

$$Q_0 \left( \frac{\omega}{k} \right) = \frac{1}{2} \left[ \ln \left| \frac{\omega+k}{\omega-k} \right| - i\pi \Theta(k^2 - \omega^2) \right] . \quad (3.9)$$

Here,  $M_{eff} = gT/\sqrt{8}$  is the effective fermion thermal mass induced by the gauge coupling.

The collective excitations correspond to the poles of the fermion propagator: there are two branches for positive energy (and two for negative energy) which are the solutions of  $* \Delta_{\pm}^{-1}(\omega, \vec{k}) = 0$ , the Dirac spinors associated with these solutions have opposite ratios of chirality to helicity  $\chi$  and the residues at the poles are  $Z_{\pm}(k)$ , respectively, with [13–18]

$$* \Delta_{\pm}^{-1}(\omega, \vec{k}) = 0 \implies \omega = \omega_{\pm}(k) , \quad Z_{\pm}(k) = \frac{1}{2M_{eff}^2} [\omega_{\pm}^2(k) - k^2] , \quad (3.10)$$

where the dispersion relations  $\omega_{\pm}(k)$  are shown in Fig. 1. The upper branch corresponds to  $\omega_+(k)$  and describes collective excitations with ratio of chirality to helicity  $\chi = +1$ ; the lower branch corresponds to  $\omega_-(k)$  describing collective excitations with ratio of chirality to helicity  $\chi = -1$ . The upper branch corresponds to the usual fermionic excitation, whereas the lower branch has vanishing group velocity (the minimum in the dispersion relation) at  $k_{min} \approx 0.4 M_{eff}$  and describes a new collective excitation in the medium, it has been named the plasmino to emphasize that it is a fermionic excitation that only exists as a collective excitation in the plasma [13–15,18]. The vanishing group velocity at  $k_{min}$  is a novel feature of the plasmino collective excitation and it will be important for the damping rate and mean free path.

We now collect some of the relevant properties of these solutions which will be relevant for the interpretation of the results (see Ref. [18] for details)

$$\begin{aligned}
k \ll M_{eff} : \omega_{\pm}(k) &\approx M_{eff} \pm \frac{k}{3}, \quad Z_{\pm}(k) \approx \frac{1}{2} \pm \frac{k}{3M_{eff}}, \\
k \gg M_{eff} : \omega_{+}(k) &\approx k + \frac{M_{eff}^2}{k}, \quad Z_{+}(k) \approx 1 + \frac{M_{eff}^2}{2k^2} \left( 1 - \ln \frac{2k^2}{M_{eff}^2} \right), \\
\omega_{-}(k) &\approx k + \frac{2k}{g} \exp \left( -\frac{2k^2}{M_{eff}^2} \right), \quad Z_{-}(k) \approx \frac{2k^2}{gM_{eff}^2} \exp \left( -\frac{2k^2}{M_{eff}^2} \right).
\end{aligned} \tag{3.11}$$

Two noteworthy aspects of these expressions will be important for the interpretation of particular features of the damping rates: (i) the dispersion relations are always above the light cone, (ii) the wave function renormalization (residue at the pole)  $Z_{-}(k)$  vanishes very fast for  $k \geq k_{min}$ . Thus the collective excitation with  $\chi = -1$  only contributes in the region of very soft external momentum  $k \leq k_{min}$ .

## B. Perturbative corrections: the damping rates

Neglecting the fermion vacuum mass term, the general form of the self-energies is dictated by rotational symmetry, hence  ${}^*\tilde{\Sigma}_{r\vec{k}}(s)$  and  $\tilde{\Sigma}_{r\vec{k}}^y(s)$  can be written as

$${}^*\tilde{\Sigma}_{r\vec{k}}(s) = -is\gamma_0 {}^*\tilde{\Sigma}_{\vec{k}}^{(0)}(s) + \vec{\gamma} \cdot \vec{k} {}^*\tilde{\Sigma}_{\vec{k}}^{(1)}(s), \tag{3.12}$$

$$\tilde{\Sigma}_{r\vec{k}}^y(s) = i\gamma_0 s \tilde{\varepsilon}_{\vec{k}}^{(0)}(s) + \vec{\gamma} \cdot \vec{k} \tilde{\varepsilon}_{\vec{k}}^{(1)}(s), \tag{3.13}$$

where we have used the notation of Ref. [29] for  $\tilde{\Sigma}_{r\vec{k}}^y(s)$ . The analytic continuation of these contributions in the complex  $s$ -plane are defined by

$${}^*\Sigma_{r\vec{k}}(\omega) \equiv {}^*\tilde{\Sigma}_{r\vec{k}}(s = -i\omega \pm 0^+) = {}^*\Sigma_{R\vec{k}}(\omega) \pm i {}^*\Sigma_{I\vec{k}}(\omega). \tag{3.14}$$

$$\Sigma_{r\vec{k}}^y(\omega) \equiv \tilde{\Sigma}_{r\vec{k}}^y(s = -i\omega \pm 0^+) = \Sigma_{R\vec{k}}^y(\omega) \pm i \Sigma_{I\vec{k}}^y(\omega). \tag{3.15}$$

To order  $g^2$  and  $y^2$ , the full fermion propagator now reads

$$S(\omega, \vec{k}) = \frac{1}{2} \left[ \overline{\Delta}_{+}(\omega, \vec{k})(\gamma_0 - \vec{\gamma} \cdot \hat{k}) + \overline{\Delta}_{-}(\omega, \vec{k})(\gamma_0 + \vec{\gamma} \cdot \hat{k}) \right], \tag{3.16}$$

where

$$\overline{\Delta}_{\pm}^{-1}(\omega, \vec{k}) = {}^*\Delta_{\pm}^{-1}(\omega, \vec{k}) + \Pi_{\pm}(\omega, \vec{k}), \tag{3.17}$$

$$\Pi_{\pm}(\omega, \vec{k}) = \frac{1}{4} \text{Tr} \left[ (\gamma_0 \mp \vec{\gamma} \cdot \hat{k}) \left( {}^*\Sigma_{r\vec{k}}(\omega) + \Sigma_{r\vec{k}}^y(\omega) \right) \right]. \tag{3.18}$$

with  ${}^*\Delta_{\pm}^{-1}(\omega, k)$  given by Eq. (3.8) and the real and imaginary parts of  $\Pi_{\pm}(\omega)$  can be read off from equations (3.14)-(3.15).

The poles of the full fermion propagator determine the excitations in the medium. The position of the poles are obtained from the zeros of  $\overline{\Delta}_{\pm}^{-1}(\omega, \vec{k})$  for  $\omega = \omega_p(k) - i\Gamma(k)$ . In the narrow width approximation  $\Gamma(k) \ll \omega_p(k)$ , the position of the complex poles are determined by the following equation

$${}^*\Delta_{\pm}^{-1}(\omega_p(k), \vec{k}) + \Pi_{\pm, R}(\omega_p(k), \vec{k}) - i \left[ Z_{\pm}^{-1}(\omega_p(k)) \Gamma(k) + \text{sgn}(\Gamma(k)) \Pi_{\pm, I}(\omega_p(k), \vec{k}) \right] = 0, \tag{3.19}$$

where we have made use of Eqs. (3.14) and (3.15), and

$$Z_{\pm}(\omega_p(k)) = \left[ \frac{\partial {}^*\Delta_{\pm}^{-1}(\omega, \vec{k})}{\partial \omega} \right]_{\omega=\omega_{\pm}(k)}^{-1}$$

are the residues at the poles for the collective excitations.

The real and imaginary parts of the above equation read

$$*\Delta_{\pm}^{-1}(\omega_p(k), \vec{k}) + \Pi_{\pm,R}(\omega_p(k), \vec{k}) = 0 , \quad (3.20)$$

$$\Gamma(k) + \text{sgn}(\Gamma(k)) Z_{\pm}(\omega_p(k)) \Pi_{\pm,I}(\omega_p(k), \vec{k}) = 0 . \quad (3.21)$$

To leading order in HTL [i.e., neglecting  $\Pi_{\pm,R}$  in Eq. (3.20)] the real part of the poles are given by the dispersion relations of collective excitations

$$\omega_p(k) = \begin{cases} \pm\omega_+(k) & \text{for fermion ,} \\ \pm\omega_-(k) & \text{for plasmino .} \end{cases} \quad (3.22)$$

The solution for the imaginary part is obtained by replacing  $\omega_p(k)$  with (3.22) in  $\Pi_{\pm,I}(\omega_p(k), \vec{k})$  to this order.

However, the equation for the imaginary part (3.21) does not have a solution because  $\Pi_{\pm,I}(\omega_p(k), \vec{k}) > 0$  (see Eqs. (3.26), (3.34) and the discussion below it) and  $Z_{\pm}(\omega_p(k)) > 0$ . Therefore, there is *no* complex pole in the physical sheet. This is a well-known result: if the imaginary part of the self-energy on shell is positive there is no complex pole solution in the *physical* sheet, the pole has moved off into the unphysical (second or higher) sheet. If the imaginary part on shell is negative, there are *two* complex poles in the physical sheet corresponding to a growing and a decaying solution, i.e., an instability. It will become clear below that  $\Pi_{\pm,I}(\omega_p(k), \vec{k}) > 0$  corresponding to a complex pole in an unphysical sheet. In this case the spectral density features a Breit-Wigner resonance shape with a width given by the damping rates

$$\Gamma_{\pm}(k) = Z_{\pm}(k) \Pi_{\pm,I}(\omega_{\pm}(k), \vec{k}) , \quad (3.23)$$

which in general determines an exponential fall-off of the fermion propagator in real time: the amplitude for collective excitations with  $\chi = \pm 1$  fall off as  $e^{-\Gamma_{\pm}(k)t}$ , an interpretation borne out by the real time evolution of the initial value problem [29,36,37]. An important exception to this analysis appears in the Abelian gauge theory because of infrared divergences in the damping rate [26,27]. These will be analyzed in detail below.

### 1. Gauge boson contribution

We now focus on the contribution to  $\Gamma_{\pm}(k)$  from the soft loop momentum region of the gauge boson exchange.

For soft momenta  $k \approx gT$ , the non-HTL contribution to the fermion self-energy from the gauge boson, i.e.,  $*\Sigma_{r\vec{k}}(\omega)$  requires HTL resummed internal propagators and vertices and is subleading by one power of  $g$ , a thorough study is presented in [23,24]. The main ingredients for a consistent computation of the non-HTL contribution are: (i) the HTL resummed internal propagators, (ii) the HTL resummed vertices [23–26]. The resummed propagators are obtained from the spectral representation in terms of the HTL spectral densities both for fermions and gauge bosons (longitudinal and transverse). When all of the lines at the vertex are soft the differential form of the Ward identities from the HTL resummed fermion self-energy can be used to obtain the resummed vertex [23–26]. We refer the reader to Refs. [23–26] for the details.

The analysis leading to the expression of the damping rates Eq. (3.23) relies on the existence and smallness of the imaginary parts of the self-energy on the mass shell of the collective excitations. However in gauge theories potential finite temperature infrared divergences could invalidate these conclusions. The infrared divergences associated with the exchange of longitudinal gauge bosons are a result of small angle Coulomb scattering and at zero temperature are those of Rutherford scattering. At finite temperature the longitudinal gauge boson (instantaneous Coulomb interaction) is screened by the Debye screening mass  $m_D \propto gT$  which cuts off the infrared and leads to a finite contribution to the damping rate from longitudinal gauge bosons. For soft external momentum this contribution has been found to be given by [24,26]

$$\Gamma^{l,g}(k) = \alpha A T , \quad (3.24)$$

with  $A$  a constant that can be found numerically [24] and in QCD is of  $\mathcal{O}(1)$  and  $\alpha = g^2/4\pi$ .

For a fermion excitation at *rest* (fermions and plasminos coincide for  $k = 0$ ), the damping rate has been computed [23–25] and found to be given by

$$\Gamma^g(k=0) = \alpha C T , \quad (3.25)$$

with  $C$  again being a numerical constant of  $\mathcal{O}(1)$  [23,24]. The reason that we do not specify the constants quantitatively is because we are interested in the damping rate for *moving* collective excitations. When the collective excitations are at rest there is no difference in their dispersion relations and therefore their transport properties are identical.

In QCD (and also in the SU(2) sector of the Standard Model) the potential infrared divergence arising from the exchanged transverse gluon propagator is conjectured to be screened by the non-perturbative magnetic screening mass  $m_{mag} \approx g^2 T$  ( $g$  is the QCD coupling constant) and the longitudinal gluon is Debye screened with  $m_D \approx g T$ . Thus the infrared divergences are cured by electric and magnetic screening lengths. The detailed analysis of Refs. [23–25] lead to the result that the damping rate of moving quasiparticles with momenta  $k \gg g^2 T$  in a non-abelian plasma are given by (up to an overall constant that depends on the gauge group structure)

$$\Gamma_{\pm}^g(k) = \frac{g^2 T}{4\pi} |v_{\pm}(k)| \ln \frac{1}{g} , \quad (3.26)$$

where  $v_{\pm}(k)$  are the group velocities for the fermion and plasmino branches. This expression is *not* valid for  $k = 0$  where the damping rates of fermion and plasmino at rest do not have the logarithmic behavior in terms of the gauge coupling [23,24], nor near  $k = k_{min}$  where the group velocity of the plasmino branch vanishes [38].

We point out that our results for the mean free paths of the collective excitations and their *difference* will be restricted to the domain of validity of Eq. (3.26), furthermore we emphasize that this regime of validity has been pointed out in Refs. [24–26,38]). In the region where the plasmino group velocity vanishes there are contributions to its damping rate that must be understood in detail, such a task is clearly beyond the scope of this article.

Since the HTL structure in QED is similar (up to gauge group factors) to that of QCD (and scalar QED), Pisarski [24] suggested that a similar form of the damping rate should be valid in a QED plasma, despite the fact that there is no magnetic screening in the Abelian theory.

In QED the transverse photon propagator is only *dynamically* screened via Landau damping and the infrared divergences remain, possibly to all orders in perturbation theory. These divergences led to questioning the quasiparticle interpretation of moving charged excitations [39,40]. More recently [26] a detailed study of the fermion propagator in the Bloch-Nordsieck (eikonal) approximation in *real time* revealed that for  $\omega_D t |v_{\pm}(k)| \gg 1$

$$S_k(t) \approx \exp[-\alpha T |v_{\pm}(k)| t \ln(\omega_D t |v_{\pm}(k)|)] , \quad (3.27)$$

with  $\omega_D \approx g T$  the Debye frequency and  $v_{\pm}(k)$  the group velocity of the fermion and plasmino branches. Although this is not an exponential relaxation that would emerge from a Breit-Wigner resonance shape of the spectral density as argued above, it does reveal a particular time scale from which a damping rate can be extracted and is given by

$$\Gamma_{\pm}^g(k) \approx \alpha T |v_{\pm}(k)| \ln \frac{1}{g} . \quad (3.28)$$

This result has also been recently found in scalar QED with an alternative method based on the renormalization group and within the context of an initial value problem as followed here [27]. Since SQED, QED and QCD share the same HTL structure to lowest order [28,18], the analysis via the renormalization group furnishes an independent confirmation of the eikonal approach.

At this stage it is important to describe the physics that leads to the damping rate from the gauge boson contribution, which as it will be seen below is rather different from that of the scalar. The constraints from energy momentum conservation in the imaginary part of the self-energy can only be satisfied below the light cone where the HTL resummed gauge boson propagator has support that arises from the Landau damping cut. The infrared process that leads to the non-exponential relaxation is the emission and absorption of soft photons or gluons at almost right angles with the moving fermion [24].

Thus we summarize the contribution to the damping rate of the collective excitations by the HTL resummed gauge boson exchange:

$$\Gamma_{\pm}^g(k) \approx \alpha T |v_{\pm}(k)| \ln \frac{1}{g} \quad \text{for } k \gg g^2 T , \quad (3.29)$$

$$\Gamma_{\pm}^g(k) \approx \alpha T \quad \text{for } k = 0 . \quad (3.30)$$

The damping rate for moving fermions is not yet available for the whole range of group velocities and clearly a better understanding of the infrared region must be pursued. In particular Eq. (3.29) is not valid near the plasmino minimum when the group velocity vanishes, since subleading contributions become important in this region [38]. Fig. 2 displays  $\Gamma_{\pm}^g(k)/M_{eff}$  vs  $k/M_{eff}$  for  $g = 0.3$ .

## 2. Scalar contribution

In Ref. [29] the damping rate for a fermion in a scalar plasma was derived. There it was found that the *decay* of the scalar into fermion-antifermion pairs results in a quasiparticle width for the fermion.

Here we study a different aspect, that is the lifetime of the *collective excitations* both fermions and plasminos due to the process of scalar decay into the collective excitations. This is one of the novel contributions of this article.

Writing the Laplace transform of the scalar contribution to the self-energy as in Eq. (3.13), the scalar contribution to  $\Pi_{\pm}(\omega, k)$  in Eq. (3.18) are given by [see Eq. (3.13) for the definition of the  $\varepsilon_{\vec{k}}^{(i)}(\omega)$ ]

$$\Pi_{\pm}^s(\omega, k) = \omega \varepsilon_{\vec{k}}^{(0)}(\omega) \pm k \varepsilon_{\vec{k}}^{(1)}(\omega) . \quad (3.31)$$

The full expressions for the  $\varepsilon_{\vec{k}}^{(i)}(\omega)$  to one loop are given in Ref. [29], we only quote here the most relevant features of their imaginary parts to clarify our arguments. The imaginary parts of these coefficients are given by

$$\begin{aligned} \varepsilon_{I, \vec{k}}^{(0)}(\omega) &= \frac{\pi}{2|\omega|} \text{sgn}(\omega) \left[ \rho_{\vec{k}}^{(0)}(|\omega|) + \rho_{\vec{k}}^{(0)}(-|\omega|) \right] , \\ \varepsilon_{I, \vec{k}}^{(1)}(\omega) &= \frac{\pi}{2} \text{sgn}(\omega) \left[ \rho_{\vec{k}}^{(1)}(|\omega|) - \rho_{\vec{k}}^{(1)}(-|\omega|) \right] , \end{aligned} \quad (3.32)$$

in terms of the following one-loop spectral densities [29]

$$\begin{aligned} \rho_{\vec{k}}^{(0)}(\omega) &= y^2 \int \frac{d^3 q}{(2\pi)^3} \frac{\bar{\omega}_q}{2\omega_{k+q}\bar{\omega}_q} \\ &\quad \times [\delta(\omega - \omega_{k+q} - \bar{\omega}_q)(1 + n_{k+q} - \bar{n}_q) + \delta(\omega - \omega_{k+q} + \bar{\omega}_q)(n_{k+q} + \bar{n}_q)] , \\ \rho_{\vec{k}}^{(1)}(\omega) &= y^2 \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2\omega_{k+q}\bar{\omega}_q} \frac{\vec{k} \cdot \vec{q}}{k^2} \\ &\quad \times [\delta(\omega - \omega_{k+q} - \bar{\omega}_q)(1 + n_{k+q} - \bar{n}_q) - \delta(\omega - \omega_{k+q} + \bar{\omega}_q)(n_{k+q} + \bar{n}_q)] , \end{aligned} \quad (3.33)$$

where  $\omega_{k+q} = \sqrt{m^2 + (\vec{k} + \vec{q})^2}$ , and  $n_{k+q}$  and  $\bar{n}_q$  are the respective distribution functions for the scalar and the fermion in the loop. The terms that contain the  $\delta(\omega - \omega_{k+q} + \bar{\omega}_q)$  do not give a contribution to the damping rate. These arise from the processes  $\psi \rightarrow \phi + \psi$  (with  $\psi$  and  $\phi$  denoting the fermion and the scalar, respectively) and result in the usual two particle cuts with support for  $|\omega| > \sqrt{k^2 + (m + M)^2}$  (here,  $M$  and  $m$  are the respective masses of the fermion and the scalar in the loop). Thus we only consider the terms proportional to  $n_{k+q} + \bar{n}_q$  in  $\Pi_{\pm, I}^s(\omega, \vec{k})$  since only the delta functions that multiply these terms will have support on the mass shell of the collective excitations as explained below. These terms lead to the following imaginary part of the scalar contribution

$$\begin{aligned} \Pi_{\pm, I}^s(\omega, \vec{k}) &= \pi y^2 \int \frac{d^3 q}{(2\pi)^3} \frac{n_{k+q} + \bar{n}_q}{2\bar{\omega}_q 2\omega_{k+q}} \left[ \left( \bar{\omega}_q \mp \text{sgn}(\omega) \hat{k} \cdot \vec{q} \right) \delta(|\omega| - \omega_{k+q} + \bar{\omega}_q) \right. \\ &\quad \left. + \left( \bar{\omega}_q \pm \text{sgn}(\omega) \hat{k} \cdot \vec{q} \right) \delta(|\omega| + \omega_{k+q} - \bar{\omega}_q) \right] . \end{aligned} \quad (3.34)$$

It is straightforward to show that  $\Pi_{\pm, I}^s(\omega, \vec{k}) > 0$  from the energy conservation constraints:  $|\omega| \pm \bar{\omega}_q \mp \omega_{k+q} = 0$ , and therefore this imaginary part is not associated with an instability but with a true resonance, i.e., a complex pole in an unphysical sheet and its real time interpretation corresponds to an exponential fall off of the amplitude of the collective excitation.

The first delta function determines a cut in the region  $0 < |\omega| < \sqrt{k^2 + (m - M)^2}$  and originates in the physical process  $\phi \rightarrow \psi + \bar{\psi}$  whereas the second delta function determines a cut in the region  $0 < |\omega| < k$  and originates in the process  $\phi + \psi \rightarrow \psi$ . The first cut describes the decay of the scalar into fermion-antifermion pairs, the second cut for ( $\omega^2 < k^2$ ) is associated with Landau damping. Both delta functions restrict the range of the integration variable  $q$  (see below). However, since the dispersion relations for the collective excitations are always above the light cone, i.e.,  $\omega_{\pm}(k) > k$ , the contribution from the second delta function vanishes on the mass shell of the collective excitations. Therefore only the first term in Eq. (3.34) contributes to the lifetime of the collective excitations.

It is at this stage that we are in position to formalize the arguments presented in the previous section to justify a one-loop computation of the scalar contribution to the self-energy of the collective excitations in terms of *free field* propagators (non HTL resummed).

The argument hinges upon two important features of the spectrum of the fermionic collective excitations: (i) The dispersion relation is always above the light cone, this feature guarantees that the second cut in (3.34) with support below the light cone will not contribute to the imaginary part evaluated on the mass-shell of the collective excitation. (ii) Under the assumption that the scalar mass  $m \gg gT$  and an external momentum  $k \approx gT$  and therefore  $|\omega| \approx gT$  on the mass shell of the collective excitations, the first delta function is satisfied only for  $\bar{\omega}_q \approx \sqrt{q^2 + m^2} \gg gT$ , hence the fermion in the loop is the ordinary single particle fermionic excitation, *not* the collective excitations associated with soft momenta. This is clear from the properties of dispersion relations and residues for the collective excitations displayed in Eq. (3.11) since for large momenta only the fermion branch survives (the wave function renormalization of the plasmino branch vanishes) and approaches the vacuum fermion dispersion relation. Hence in summary: the contribution from scalar exchange to damping rates of the soft collective excitations is obtained from the one loop self-energy in which the internal scalar *and* fermion lines are the free particle propagators, but the external line are the fermion or plasmino collective excitations.

This is a remarkable result, *the fermions and plasminos acquire a width through the induced decay of the scalar in the medium* and the damping rates for the soft collective excitations are *different*. For large scalar mass  $m \gg gT$  the kinematics dictates that the scalar decays into a soft collective excitation, either fermion or plasmino, and a hard fermion.

In the limit of  $M \ll gT$ , we finally find

$$\Pi_{\pm,I}^s(\omega, \vec{k}) = \frac{y^2}{32\pi k^2} \int_{q_1^*(\omega)}^{q_2^*(\omega)} dq (n_{k+q} + \bar{n}_q) [2kq \mp \text{sgn}(\omega) (2|\omega|q + \omega^2 - \omega_k^2)] \quad (3.35)$$

for  $|\omega| > k$ , where  $\omega_k = \sqrt{m^2 + k^2}$  and

$$q_1^*(\omega) = \frac{1}{2} \left| \frac{\omega^2 - \omega_k^2}{|\omega| + k} \right|, \quad q_2^*(\omega) = \frac{1}{2} \left| \frac{\omega^2 - \omega_k^2}{|\omega| - k} \right|.$$

From the expression for the total damping rates Eq. (3.23), the scalar contribution to the damping rates for the fermion and plasmino collective excitations is found by evaluating the scalar contribution of the imaginary part of the self-energy (3.34)-(3.35) on the mass shell of the collective modes. We find the damping rates to be given by

$$\begin{aligned} \Gamma_{\pm}^s(k) &= Z_{\pm}(k) \Pi_{\pm,I}^s(\omega_{\pm}(k), \vec{k}) \\ &= \frac{y^2 Z_{\pm}(k)}{32\pi k^2} \left\{ \pm T (\omega_k^2 - \omega_{\pm}^2(k)) \left[ \ln(1 - e^{-\beta[\omega_{\pm}(k)+q]}) - \ln(1 + e^{-\beta q}) \right] \right. \\ &\quad \pm 2T (\omega_{\pm}(k) \mp k) \left[ \omega_{\pm}(k) \ln(1 - e^{-\beta[\omega_{\pm}(k)+q]}) + q \ln(1 + e^{-\beta q}) \right] \\ &\quad \left. \mp 2T^2 (\omega_{\pm}(k) \mp k) \left[ \text{Li}_2(1 - e^{-\beta[\omega_{\pm}(k)+q]}) + \text{Li}_2(-e^{-\beta q}) \right] \right\} \bigg|_{q_1^*(\omega_{\pm}(k))}^{q_2^*(\omega_{\pm}(k))}, \end{aligned} \quad (3.36)$$

respectively. Here,

$$\text{Li}_2(x) \equiv \int_x^0 \frac{dt}{t} \ln(1 - t)$$

is the dilogarithm function.

Although these expressions are somewhat unwieldy, there are important features that are very revealing in Eq. (3.36): these are the wave function renormalization factors  $Z_{\pm}(k)$ . As we discussed in the previous section and is clear from the expressions (3.11) the wave function renormalization for the plasmino branch vanishes very fast for  $k \geq k_{min} \approx 0.4 M_{eff}$ . The damping rates  $\Gamma_{\pm}^s(k)$  are displayed in Fig. 3 vs  $k/M_{eff}$  for  $g = 0.3$  and  $m = T$ . The upper solid line is the damping rate for the fermion branch, and the lower dashed line is the damping rate for the plasmino branch, the rapid decrease for the plasmino branch beyond  $k \geq M_{eff}$  is a result of the vanishing of the wave function renormalization.

#### IV. MEAN FREE PATHS: A WAVE PACKET INTERPRETATION

The notion of mean free paths in a plasma in which different collective excitations with different group velocities are present is rather subtle and requires a careful treatment. It is here where the real time description in terms of

the in-medium Dirac equation is fruitful. We obtain the mean free paths by studying the space-time evolution of an initially prepared wave packet. This formulation identifies unambiguously the mean free paths without resorting to a diffusion interpretation of transport.

In particular we are interested in transport of chirality by the collective excitations, which evolve independently in time. Since the collective excitations are eigenstates of the effective Dirac equation in the leading HTL order, they are given by the solutions of

$$\left[ {}^*\Delta_-^{-1}(\omega, \vec{k})(\gamma_0 - \vec{\gamma} \cdot \hat{k}) + {}^*\Delta_+^{-1}(\omega, \vec{k})(\gamma_0 + \vec{\gamma} \cdot \hat{k}) \right] \tilde{\psi}_{\vec{k}}(s = -i\omega) = 0. \quad (4.1)$$

Let us consider that the initial expectation value  $\psi_{\vec{k}}(0)$  in Eq. (2.14) is of the form

$$\psi_{\vec{k}}^{\pm}(0) \propto \mathcal{U}_{\vec{k}}^{\pm} e^{-\frac{R^2}{2}(\vec{k}-\vec{k}_o)^2}, \quad (4.2)$$

where the spinors  $\mathcal{U}_{\vec{k}}^{\pm}$  are annihilated by  $\gamma_0 \mp \vec{\gamma} \cdot \hat{k}$ , i.e., have chirality to helicity ratios  $\chi = \pm 1$  respectively. This initial state describes a wave packet of collective modes localized in momentum at  $\vec{k}_o$ . In space it describes a wave packet localized over a spatial extent  $R$  around the origin with expectation value of the momentum  $\vec{k}_o$ . To describe an almost monochromatic beam of collective excitations we will choose  $R \gg \omega_p^{-1}$  since  $\omega_p^{-1}$  is the typical spatial extent of the quasiparticle states (screening cloud). The quasiparticle is described by a spectral function of the Breit-Wigner form (although this is not evident for the logarithmic relaxation in QED, it has been argued to be a good approximation in this case [26]). The time evolution can then be obtained by inverse Laplace transform, the details of which can be found in Ref. [37]. In the narrow width approximation  $\Gamma_{\pm}^t(k) \ll \omega_{\pm}(k)$ , we find the following time evolution of the initial wave packet

$$\psi^{\pm}(\vec{x}, t) \approx \int d^3k e^{i\vec{k} \cdot \vec{x}} Z_{\pm}(k) \mathcal{U}_{\vec{k}}^{\pm} e^{-\frac{R^2}{2}(\vec{k}-\vec{k}_o)^2} e^{-i\omega_{\pm}(k)t} e^{-\Gamma_{\pm}^t(k)t}, \quad (4.3)$$

where  $\Gamma_{\pm}^t(k)$  is the *total* damping rate, i.e., gauge plus scalar contribution and  $Z_{\pm}(k)$  the residue at the quasiparticle pole (wave function renormalization). Since the initial wave packet is strongly peaked in momentum, we can perform the integral over momentum by expanding  $\omega_{\pm}(k)$  and  $\Gamma_{\pm}^t(k)$  around  $k = k_o$  and using the narrow width approximation. We find

$$\psi^{\pm}(\vec{x}, t) \propto Z_{\pm}(k_o) \mathcal{U}_{\vec{k}_o}^{\pm} e^{-\frac{\vec{x}_{\perp}^2(t)}{R^2(t)}} e^{-\Gamma_{\pm}^t(k_o)t} e^{i\vec{k}_o \cdot (\vec{x} - \vec{V}_{p,\pm} t)}, \quad (4.4)$$

where

$$\vec{X}_{\pm}(t) = \vec{x} - \vec{V}_{g,\pm} t, \quad R^2(t) = R^2 + i \left. \frac{d^2 \omega_{\pm}(k)}{dk^2} \right|_{k_o} t, \quad (4.5)$$

$$\vec{V}_{g,\pm} = \hat{k}_o \left. \frac{d\omega_{\pm}(k)}{dk} \right|_{k_o} = \hat{k}_o v_{\pm}(k_o), \quad (4.6)$$

$$\vec{V}_{p,\pm} = \hat{k}_o \frac{\omega_{\pm}(k_o)}{k_o}. \quad (4.7)$$

Obviously  $\vec{V}_{g,\pm}$  and  $\vec{V}_{p,\pm}$  are the group and phase velocities, respectively. We then see that the peak amplitude of the wave packet attenuates in space on a distance scale given by the *mean free paths*

$$\lambda_{\pm}(k_o) = \frac{|v_{\pm}(k_o)|}{\Gamma_{\pm}^t(k_o)}, \quad (4.8)$$

where  $v_{\pm}(k)$  are the group velocities for the fermion and plasmino branches and we took the absolute value because there is a region of negative group velocity for the plasmino branch (see Fig. 1) before the minimum at  $k \approx 0.4 M_{eff}$ . Eq. (4.8) is the main reason for the wave packet analysis: it unambiguously identifies the mean free paths of the collective excitations in terms of the group velocities and damping rates for each branch of collective excitation independently.

Since the gauge contribution to the damping rates of collective excitations is proportional to their group velocities (see Eq. (3.29)) we then highlight the fact that *whereas the gauge contribution to the damping rates of the collective*

modes are different, their mean free paths are the same. Hence the *difference of mean free paths* which is the goal of our study is solely given (to this order) by the scalar contribution. This is the main result of this study.

Using the expression for the gauge contribution to the damping rate of moving quasiparticles given by Eq. (3.29), we can write the mean free paths in the following form

$$\lambda_{\pm}(k) = \frac{1}{\Gamma_o^g} \left[ 1 + \frac{\Gamma_{\pm}^s(k)}{|v_{\pm}(k)| \Gamma_o^g} \right]^{-1}, \quad \Gamma_o^g \approx \alpha T \ln \frac{1}{g}. \quad (4.9)$$

This expression reveals clearly that the *difference* of mean free paths for the fermion and plasmino collective excitations is mostly given by the contribution to the damping rate from the *scalar sector*. This is one of the important results in this article that we want to emphasize: although the largest contribution to the damping rates for soft collective excitations arises from the gauge boson contribution to the self-energy, since  $\alpha \gg y^2$ , the *difference* of mean free paths is determined mainly by the decay of the heavy scalar. Fig. 4 shows  $[\lambda_+(k) - \lambda_-(k)]M_{eff}$  vs  $k/M_{eff}$  for  $g = 0.3$ ,  $y = 10^{-4}$  and  $m = T$ . This figure displays one of the important results of this article. The strong peak near the plasmino minimum is a result of the vanishing of the plasmino group velocity and as discussed previously the gauge contribution to the damping rate of the plasmino branch is not reliable near this region but is so beyond and before this point.

In particular away from the minimum from the plasmino branch and using  $y^2 \ll \alpha$ , the difference in mean free paths is approximately given by

$$\lambda_+(k) - \lambda_-(k) \approx \frac{1}{(\Gamma_o^g)^2} \left[ \frac{\Gamma_-^s(k)}{|v_-(k)|} - \frac{\Gamma_+^s(k)}{v_+(k)} \right]. \quad (4.10)$$

Thus the fermion and plasmino soft collective excitations have different mean free paths and the difference is determined mostly by the heavy scalar decay into a hard and a soft collective excitation.

## V. SUMMARY, IMPLICATIONS FOR THE STANDARD MODEL AND CONCLUSIONS

We have focused our attention on the transport properties of soft collective excitations in an abelian gauge, fermion and scalar plasma with the goal of understanding chirality transport as relevant for non-local baryogenesis.

Our main observation is that the transport properties of *soft* collective excitations are different for the different branches of the fermionic dispersion relation. Since these collective excitations carry ratios of chirality to helicity  $\chi = \pm 1$ , different transport coefficients, i.e., damping rates and mean free paths for these excitations will result in a differential transport of chirality. There are two main physical mechanisms that lead to different transport properties: (i) The absorption and emission of soft gauge bosons that are dynamically screened by Landau damping on scales  $gT$  and by a magnetic mass for the non-abelian sector on scales  $g^2T$ . (ii) The *decay* of the heavy scalar (Higgs) particle into hard fermion and a soft fermion or plasmino results in a contribution to the damping rate of on-shell collective excitations. The physical aspects of these two phenomena are fairly robust, in particular the hard thermal loop structure of the fermionic self-energy has the *same form* in the abelian (QED) and non-abelian (SM) theories, with the only differences being in the overall scales (see for example Ref. [8] wherein the contributions of the weak and scalar sectors to the leading HTL fermion self-energy had been computed in detail). The decay of a heavy Higgs into a hard fermion and a soft collective excitation is a consequence of simple kinematics in the heat bath and is therefore fairly independent of the model. Obviously the strength of the Yukawa couplings and the group representation of the scalar fields will change the overall normalization of the scalar contribution.

Therefore we believe that the results obtained in this article are relevant for non-local baryogenesis in the Standard Model or extensions thereof and that the main ingredients that determine the different transport coefficients described in this study are fairly robust.

We have combined detailed studies of the damping rates of moving fermionic quasiparticles [23–26] and extended the recently studied phenomenon of fermionic damping rates via heavy scalar decay [29] to provide a description of the damping rates and mean free paths of collective excitations with momenta  $k \gg g^2T$  to leading order in Hard Thermal Loop resummation and lowest order in Yukawa coupling. This analysis is valid for the lightest quarks and leptons in the Standard Model (certainly *not* the top quark) and both in the symmetric and broken symmetry phases, since for the lightest fermions  $gT \gg M$  with  $M$  the vacuum masses. The main uncertainties in our results stem from the infrared sensitivity of the gauge contributions to the fermionic damping rate, in particular the magnetic sector and its non-perturbative screening scale. This is still an ongoing subject of active study [24–26] and certainly beyond the scope of this article. However as argued persuasively in [24–26] the results given by Eqs. (3.28) are trustworthy for  $k \gg g^2T$  and away from the minimum in the plasmino branch when the plasmino group velocity vanishes. For



a first order phase transition the Higgs mass does not vary much near the phase transition from its vacuum value  $m \approx T \approx 100\text{GeV}$ . This translates into that the scalar contribution to the fermionic self-energy is not sensitive to the infrared behavior and does not require hard-thermal loop resummation.

By casting our studies in terms of the real time, in-medium effective Dirac equation, we were able to study the real time evolution of wave packets of collective excitations and extract unambiguously the mean free paths or attenuation lengths for the different collective excitations. This approach transcends any approximation that relies on a diffusive description of transport.

Our main results are summarized by the expressions: (3.36) for the scalar contribution to the damping rate and (4.9) for the mean free paths for the soft collective excitations with  $k \gg g^2 T$  and away from the plasmino minimum  $k \approx 0.4 M_{eff}$ . The main observation is that the *difference* of mean free paths for the fermion and plasmino branches for  $k \gg g^2 T$  and away from the minimum of the plasmino branch is approximately given by Eq. (4.10), i.e., *mainly* determined by the different *scalar* contributions to the damping rate of the collective excitations. The uncertainties near the region of the plasmino minimum are related to the infrared sensitivity of the fermionic damping rates and require a careful analysis of the infrared region of the HTL resummed self-energies, such a study is currently under way.

There is potentially an important exception to the validity and robustness of our results in the case of extensions of the Standard Model that include scalars that are too light. Light scalars are common in supersymmetric extensions and extensions with more than one doublet. They are welcome because they tend to increase the strength of the first order phase transition [9]. If light scalars are present, their contribution to the fermionic self-energy will have to be understood in detail and assess in each particular model whether an HTL resummation of the scalar propagator as well as that of the fermion in the loop is necessary. Clearly such cases must be analyzed in detail for the particular models.

A very important limitation of our results is the knowledge of the damping rate of the plasmino near the region where its group velocity vanishes. As recognized by the authors [24–26,38] of the original work, in this region there are further contributions to the plasmino damping rate that must be understood in detail.

We think that the novel results found in this article for the transport of chirality by soft collective excitations reveal new features which are distinct from those previously obtained for hard excitations and could have an important impact in mechanisms of local baryogenesis. Whereas the main focus of this article has been a quantitative understanding of chirality transport by *soft* collective excitations and a detailed understanding of the gauge and scalar contributions to the transport coefficients, obviously the next step is to provide a quantitative assessment of the impact of these new results on the baryon asymmetry within particular extensions of the Standard Model. This study is currently underway.

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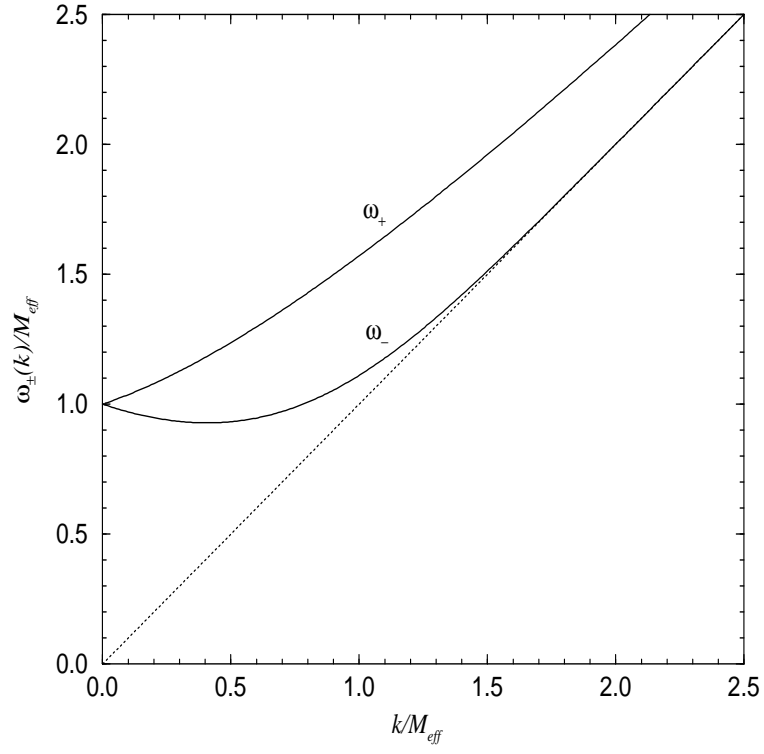


FIG. 1. Dispersion relations: the upper corresponds to the fermion branch and the lower to the plasmino branch. The dashed line is the light cone.

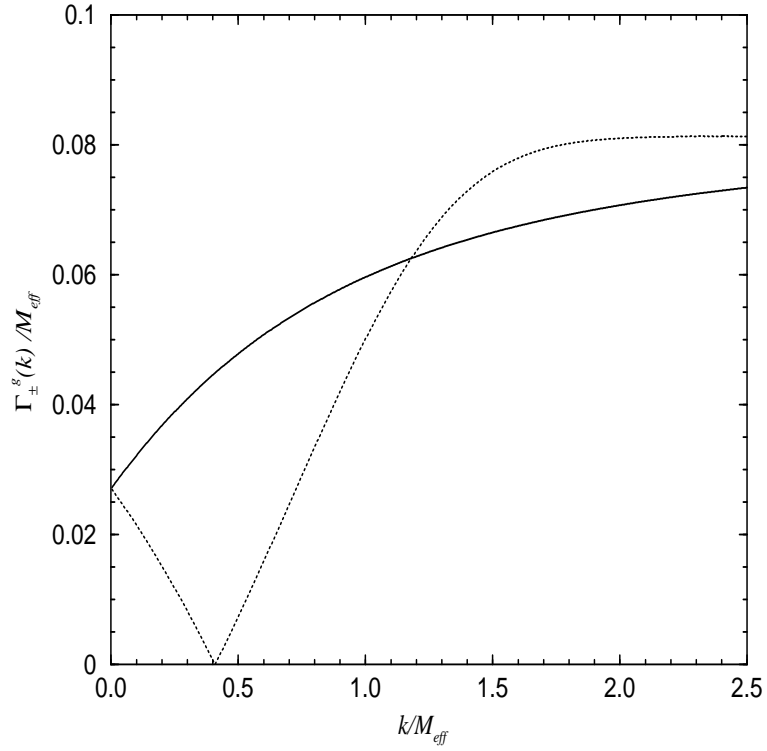


FIG. 2.  $\Gamma_{\pm}^g/M_{eff}$  vs  $k/M_{eff}$  for  $g = 0.3$ . The solid line corresponds to the fermion branch and the dashed line to the plasmino branch.

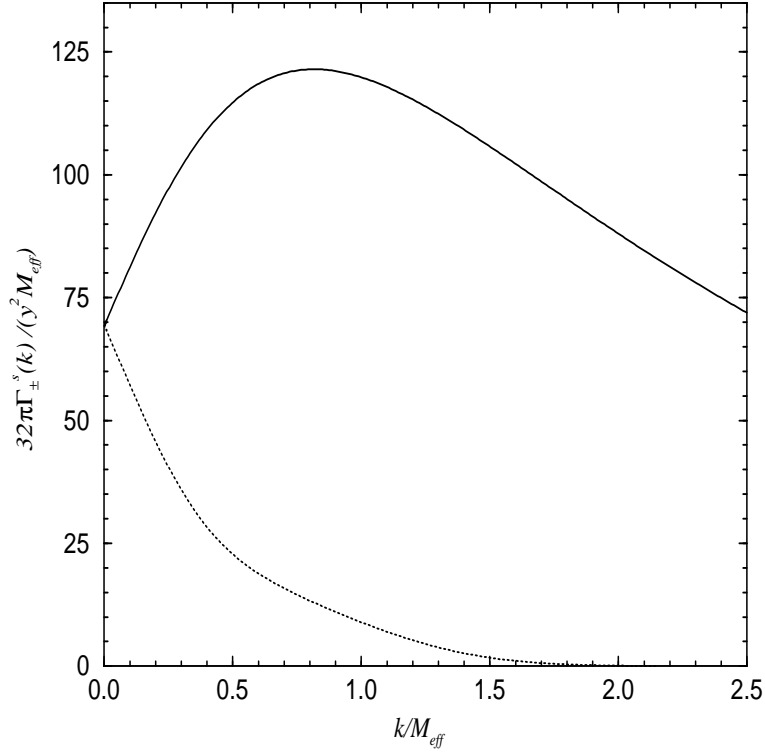


FIG. 3.  $32\pi\Gamma_{\pm}^s(k)/(y^2 M_{eff})$  vs  $k/M_{eff}$  for  $m = T$ . The solid line is for the fermion branch and the dashed line for the plasmino branch. The scalar contribution  $\Gamma_{\pm}^s(k)$  is given by Eq. (3.36) in the text.

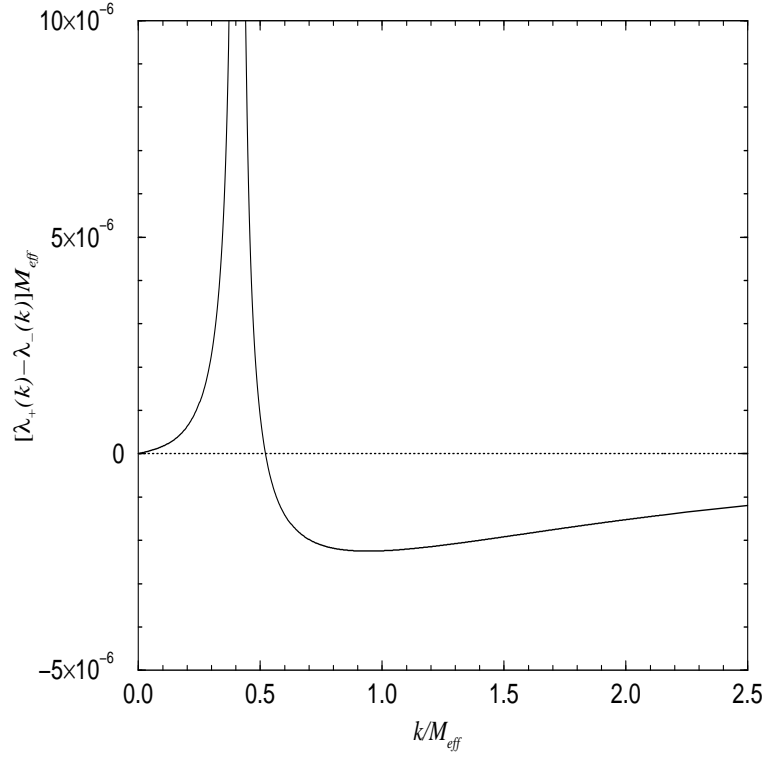


FIG. 4.  $[\lambda_+(k) - \lambda_-(k)]M_{eff}$  vs  $k/M_{eff}$  for  $g = 0.3$ ,  $y = 10^{-4}$  and  $m = T$ . The peak near  $k \approx 0.4 M_{eff}$  is a consequence of the vanishing of the plasmino group velocity, the region near this peak is not trustworthy as the expression for the gauge contribution to the damping rate of the plasmino is not valid in the region where its group velocity vanishes as explained in the text below Eq. (3.26) .